Altshuler-Aronov correction to the conductivity of a large metallic square network

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We consider the correction $\Delta\sigma_{\mathrm{ee}}$ due to electron-electron interaction to the conductivity of a weakly disordered metal (Altshuler-Aronov correction). The correction is related to the spectral determinant of the Laplace operator. The case of a large square metallic network is considered. The variation of $\Delta\sigma_{\mathrm{ee}}(L_T)$ as a function of the thermal length L_T is found very similar to the variation of the weak localization $\Delta\sigma_{\mathrm{WL}}(L_\varphi)$ as a function of the phase coherence length. Our result for $\Delta\sigma_{\mathrm{ee}}$ interpolates between the known one-dimensional (1D) and two-dimensional (2D) results, but the interaction parameter entering the expression of $\Delta\sigma_{\mathrm{ee}}$ keeps a 1D behavior. Quite surprisingly, the result is very close to the 2D logarithmic behavior already for $L_T \sim a/2$, where a is the lattice parameter.

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I. INTRODUCTION

At low temperature, the classical (Drude) conductivity of a weakly disordered metal is affected by two kinds of quantum corrections: the first one is the *weak-localization* (WL) correction, a phase coherent contribution that originates from quantum interferences between reversed electronic trajectories. This contribution to the averaged conductivity depends on the phase coherence length L_φ and the magnetic field: $\Delta\sigma_{\mathrm{WL}}(\mathcal{B},L_\varphi)$. The temperature manifests itself through L_φ, since phase breaking may depend on temperature, e.g., if it originates from electron-electron[1] or electron-phonon[2] interaction.

In a metal, an electron is not only elastically scattered on the disordered potential, but, due to the electron-electron interaction, is also scattered by the electrostatic potential created by the other electrons. At low temperatures, when the elastic scattering rate $(1/\tau_e)$ dominates the electron-electron scattering rate $[1/\tau_{\mathrm{ee}}(T)]$, the motion of the electron is diffusive between scattering events with other electrons. In this regime, electron-electron interaction is responsible for a small depletion of the density of states at Fermi energy (called the DOS anomaly or the Coulomb dip) and a correction to the averaged conductivity as well, the so-called Altshuler-Aronov (AA) correction[3–9] (see Refs. 10–12 for a recent discussion). AA and WL corrections are of the same order (but the latter vanishes in a magnetic field). However, contrary to the WL, the AA correction is not sensitive to phase coherence and involves another important length scale: the thermal length $L_T=\sqrt{D/T}$ ($\hbar=k_B=1$). The AA correction, denoted below as $\Delta\sigma_{\mathrm{ee}}(L_T)$, has been measured in metallic wires in several experiments.[14–17] From the experimental point of view, AA correction allows one to study interaction effects in weakly disordered metals, but also furnishes a local probe of temperature in order to control Joule heating effects,[15,17] which is crucial in a phase coherent experiment.

All the works aforementioned refer to the quasi-one-dimensional (wire) or two-dimensional (plane) situations. Quantum transport has also been studied in more complex geometries like networks of quasi-one-dimensional (quasi-1D) wires. For example, several studies of WL have been provided on large regular networks in honeycomb and square metallic networks,[18,19] in square networks etched in a two-dimensional electron gas,[20] and in square and dice silver networks.[21] Theoretical studies of WL on networks have been initiated by the works of Douçot and Rammal[22,23] and improved by Pascaud and Montambaux,[24] who introduced a powerful tool:[25] the spectral determinant of the Laplace operator, which will be used in the following (see also Ref. 26).

The aim of this paper is to study how the AA correction can be computed in networks. In the first part, we briefly recall how the spectral determinant can be used to compute the WL. Then in the second part, we will consider the AA correction.

II. SPECTRAL DETERMINANT AND WEAK LOCALIZATION

Interferences of reversed electronic trajectories are encoded in the Cooperon, solution of a diffusionlike equation $(\partial_t - D[\nabla - 2ieA(x)]^2)\mathcal{P}_c(x,x';t)=\delta(x-x')\delta(t)$, where $A(x)$ is the vector potential. On large regular networks, when it is justified to integrate uniformly the Cooperon over the network (see Ref. 27 for a discussion of this point), it is meaningful to introduce the space-averaged Cooperon $\mathcal{P}_c(t) = \int \frac{dx}{\mathrm{Vol}}\mathcal{P}_c(x,x;t)$, then

$$\Delta\sigma_{\mathrm{WL}} = -\frac{2e^2D}{\pi}\int_0^\infty dt\, e^{-t/\tau_\varphi}\,\mathcal{P}_c(t) \tag{1a}$$

$$= -\frac{2e^2}{\pi}\frac{1}{\mathrm{Vol}}\frac{\partial}{\partial\gamma}\ln S(\gamma), \tag{1b}$$

where $\tau_\varphi = L_\varphi^2/D$ is the phase coherence time. The factor 2 stands for spin degeneracy. We have omitted in Eqs. (1a) and (1b) a factor $1/s$, where s is the cross section of the wires. The parameter γ is related to the phase coherence length $\gamma = 1/L_\varphi^2$ (note that the description of the decoherence due to electron-electron interaction in networks requires a more refined discussion[28,29]). The spectral determinant of the Laplace operator is formally defined as $S(\gamma)=\det(\gamma-\Delta)$

$=\Pi_n(\gamma+E_n)$, where $\{E_n\}$ is the spectrum of $-\Delta$ [in the presence of a magnetic field, $\Delta\to(\nabla-2ieA)^2$]. The interest in introducing $S(\gamma)$ is that it can be related to the determinant of a $V\times V$ matrix, where V is the number of vertices that encodes all information on the network (topology, length of the wires, magnetic field, and connection to reservoirs). We label vertices by greek letters. $l_{\alpha\beta}$ designates the length of the wire $(\alpha\beta)$ and $\theta_{\alpha\beta}$ the circulation of the vector potential along the wire. The topology is encoded in the adjacency matrix: $a_{\alpha\beta}=1$ if α and β are linked by a wire, $a_{\alpha\beta}=0$ otherwise. $\lambda_\alpha=\infty$ if α is connected to a reservoir, and $\lambda_\alpha=0$ if not. We introduce the matrix

$$\mathcal{M}_{\alpha\beta}=\delta_{\alpha\beta}\left(\lambda_\alpha+\sqrt{\gamma}\sum_\mu a_{\alpha\mu}\coth\sqrt{\gamma}l_{\alpha\mu}\right)-a_{\alpha\beta}\sqrt{\gamma}\frac{e^{-i\theta_{\alpha\beta}}}{\sinh\sqrt{\gamma}l_{\alpha\beta}}, \tag{2}$$

where the $a_{\alpha\mu}$ constraints the sum to run over neighboring vertices. Then[24]

$$S(\gamma)=\prod_{(\alpha\beta)}\frac{\sinh\sqrt{\gamma}l_{\alpha\beta}}{\sqrt{\gamma}}\det\mathcal{M}, \tag{3}$$

where the product runs over all wires. We now consider a large square network of size $N_x\times N_y$ made of wires of length $l_{\alpha\beta}=a\,\forall(\alpha\beta)$. For simplicity, we impose periodic boundary conditions (topology of a torus), which is inessential as soon as the total size of the network remains large compared to L_φ. At zero magnetic field, the spectrum of the adjacency matrix is $\epsilon_{n,m}=2\cos(2n\pi/N_x)+2\cos(2m\pi/N_y)$, with $n=1,\ldots,N_x$ and $m=1,\ldots,N_y$. Therefore

$$S(\gamma)=\left(2\frac{\sinh\sqrt{\gamma}a}{\sqrt{\gamma}}\right)^{N_xN_y}\times\prod_{n=1}^{N_x}\prod_{m=1}^{N_y}\left(2\cosh\sqrt{\gamma}a-\cos\frac{2\pi n}{N_x}-\cos\frac{2\pi m}{N_y}\right). \tag{4}$$

The calculation of $\ln S(\gamma)$ involves a sum that can be replaced by an integral when $N_x,N_y\gg L_\varphi/a$. Using

$$\int_0^{2\pi}\frac{dxdy}{(2\pi)^2}\frac{1}{2A+\cos x+\cos y}=\frac{1}{\pi A}K\left(\frac{1}{A}\right), \tag{5}$$

where $K(x)$ is the complete elliptic integral of the first kind,[30] yields[20]

$$\frac{1}{\mathrm{Vol}}\frac{\partial}{\partial\gamma}\ln S(\gamma)=\frac{1}{4\sqrt{\gamma}}\left[\coth\sqrt{\gamma}a-\frac{1}{\sqrt{\gamma}a}+\frac{2}{\pi}\tanh\sqrt{\gamma}a\,K\left(\frac{1}{\cosh\sqrt{\gamma}a}\right)\right], \tag{6}$$

where the volume of the network is $\mathrm{Vol}=2N_xN_ya$. We recover the expression of the WL first derived by Douçot and Rammal.[23] Figure 1 displays the dependence of the WL correction as a function of the phase coherence length L_φ. We now discuss two limiting cases.

1D limit. In the limit $L_\varphi\ll a$ (i.e., $\sqrt{\gamma}a\gg1$),

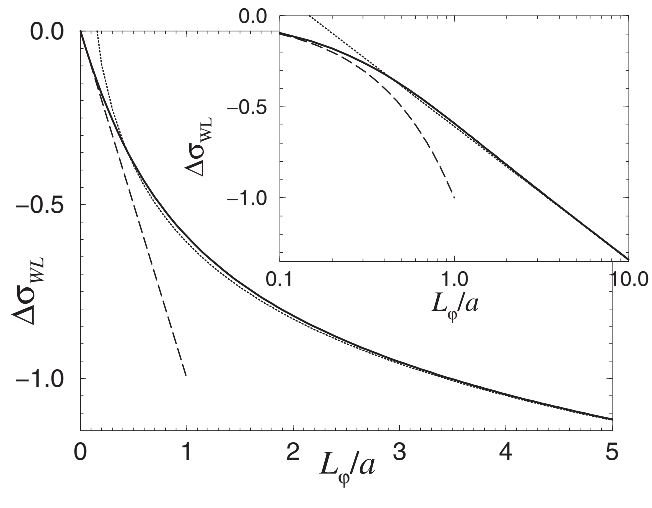

FIG. 1. $\Delta\sigma_{\mathrm{WL}}$ in unit of $2e^2/h$ as a function of L_φ/a (at zero magnetic field). The dashed line is the 1D result. The dotted line is the 2D limit [Eq. (9)].

$$\Delta\sigma_{\mathrm{WL}}=-\frac{2e^2}{h}\left[L_\varphi-\frac{L_\varphi^2}{2a}+O(e^{-2a/L_\varphi})\right]. \tag{7}$$

We compare with the result for a wire of length a connected at its extremities: $\Delta\sigma_{\mathrm{WL}}^{\mathrm{wire}}\simeq-\frac{2e^2}{h}\left(L_\varphi-\frac{L_\varphi^2}{a}\right)$. As we can see, the dominant terms coincide. Deviations appear when L_φ/a increases since trajectories begin to feel the topology of the network. This is already visible by comparing the second terms of the expansions.

2D limit. In the limit $L_\varphi\gg a$ (i.e., $\sqrt{\gamma}a\ll1$), we obtain

$$\frac{1}{\mathrm{Vol}}\frac{\partial}{\partial\gamma}\ln S(\gamma)=\frac{a}{2\pi}\left[\ln\left(\frac{4L_\varphi}{a}\right)+\frac{\pi}{6}+O\left(\frac{a^2}{L_\varphi^2}\ln\frac{L_\varphi}{a}\right)\right]. \tag{8}$$

The conductivity reads

$$\Delta\sigma_{\mathrm{WL}}\simeq-\frac{2e^2}{h}a\left[\frac{1}{\pi}\ln\left(\frac{L_\varphi}{a}\right)+C_{\mathrm{WL}}\right], \tag{9}$$

with $C_{\mathrm{WL}}=\frac{2\ln2}{\pi}+\frac{1}{6}\simeq0.608$. As noticed in the beginning of the section, Eqs. (7) and (9) should be divided by the cross section s of the wires. In the two-dimensional (2D) limit, diffusive trajectories expand over distances larger than a and feel the two-dimensional nature of the system, being the reason why Eq. (9) is reminiscent of the 2D result. It is interesting to point out that the network provides a natural cutoff (the length of the wires, a), while the computation of the WL for a plane in the diffusion approximation requires one to introduce a cutoff by hand for lower times in Eq. (1a), which is the elastic scattering time τ_e. In this latter case, the constant added to the logarithmic behavior is not well controlled since it depends on the cutoff procedure (the computation of the constant for a plane requires one to go beyond the diffusion approximation and leads to[31] $\Delta\sigma_{\mathrm{WL}}^{\mathrm{plane}}=-\frac{e^2}{\pi h}\times\ln(2L_\varphi^2/\ell_e^2+1)\simeq-\frac{2e^2}{h}\left[\frac{1}{\pi}\ln(L_\varphi/\ell_e)+\frac{1}{2\pi}\ln2\right]$ since $\ell_e\ll L_\varphi$).

III. AL'TSHULER-ARONOV CORRECTION

At first order in the electron-electron interaction, the exchange term is the dominant contribution to the correction to the conductivity[8,10,11,32]

$$\Delta\sigma_{\mathrm{ee}} = -\frac{2\sigma_0}{d\pi \mathrm{Vol}}\int_{-\infty}^{+\infty} d\omega \frac{\partial}{\partial\omega}\left(\omega \coth\frac{\omega}{2T}\right) \times \mathrm{Im}\sum_{\vec{q}} D\vec{q}^{\,2}\frac{U(\vec{q},\omega)}{(-i\omega + D\vec{q}^{\,2})^3}, \quad (10)$$

where $U(\vec{q},\omega)$ is the dynamically screened interaction. Within the random-phase approximation and in the small $\vec{q}$ and ω limits, the interaction takes the form[33] $U(\vec{q},\omega) \simeq \frac{1}{2\rho_0}\frac{-\mathrm{i}\omega + D\vec{q}^{\,2}}{D\vec{q}^{\,2}}$, where ρ_0 is the density of states per spin channel. Replacing the Drude conductivity by its expression $\sigma_0 = 2e^2\rho_0 D$ and performing an integration by parts, we get

$$\Delta\sigma_{\mathrm{ee}} = -\frac{2e^2 D}{\pi d \mathrm{Vol}}\int d\omega \frac{\partial^2}{\partial\omega^2}\left(\omega \coth\frac{\omega}{2T}\right)\mathrm{Re}\sum_{\vec{q}}\frac{1}{-i\omega + D\vec{q}^{\,2}}. \quad (11)$$

After Fourier transform, the result can be cast in the form[11]

$$\Delta\sigma_{\mathrm{ee}} = -\lambda_\sigma \frac{e^2 D}{\pi}\int_0^\infty dt \left(\frac{\pi T t}{\sinh \pi T t}\right)^2 \mathcal{P}_d(t). \quad (12)$$

For the exchange term considered here, one finds $\lambda_\sigma = 4/d$. Further calculation yields[8] $\lambda_\sigma \simeq \frac{4}{d} - \frac{3}{2}F$, where F is the average of the interaction on the Fermi surface (see definition in Refs. 8 and 9). This expression of λ_σ is valid in the perturbative regime, $F \ll 1$; nonperturbative expression is given in Refs. 6–9. $\mathcal{P}_d(t)$ is the space integrated return probability $\mathcal{P}_d(t) = \int \frac{dx}{\mathrm{Vol}}\mathcal{P}_d(x,x;t)$, where $\mathcal{P}_d(x,x';t)$ is the solution of a classical diffusion equation similar to the equation for $\mathcal{P}_c(x,x';t)$, apart that it does not feel the magnetic field: $[\partial_t - D\Delta]\mathcal{P}_d(x,x';t) = \delta(x-x')\delta(t)$. Therefore the Laplace transform of $\mathcal{P}_d(t)$ is given by $\partial_\gamma \ln S(\gamma)$ with $\theta_{\alpha\beta}=0$. It is interesting to point out that Eq. (12) has a similar structure to Eq. (1a), with a different cutoff procedure for large time. It also involves a different scale: the temperature dependence of $\Delta\sigma_{\mathrm{ee}}$ is driven by the length scale L_T instead of L_φ for the weak-localization correction $\Delta\sigma_{\mathrm{WL}}$.

Up to Eq. (12) the discussion is rather general and nothing has been specified on the system. We have seen in Sec. II that the WL for the square network presents a dimensional crossover from one dimensional to two dimensional by tuning L_φ/a. A similar dimensional crossover occurs for the AA correction by tuning L_T/a, as we will see. This remark raises the question of the dimension d in Eq. (10). To answer this question we should return to the origin of the factor $1/d$: the current lines in the conductivity σ_{ij} produce a factor $q_i q_j$ replaced by $\delta_{ij}\frac{1}{d}\vec{q}^{\,2}$ after angular integration. Since in a network the diffusion in the wires has a 1D structure (provided that $W \ll L_T \sim \sqrt{D/\omega}$, where W is the width of the wires), the dimension in λ_σ is $d=1$. Therefore we have for the network, $\lambda_\sigma^{\mathrm{network}} \simeq 4 - \frac{3}{2}F$.

If one now expands the thermal function in Eqs. (12) as

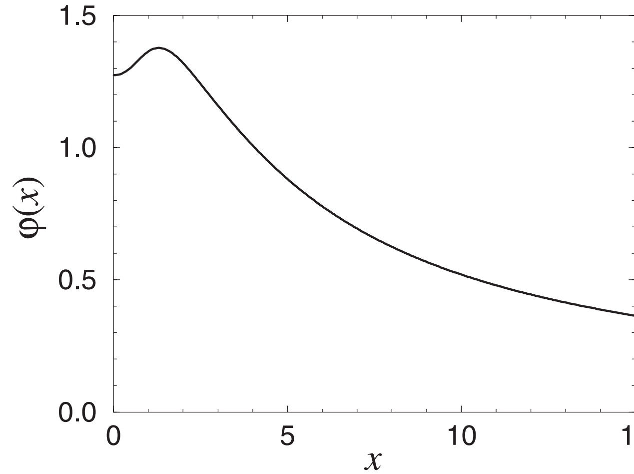

FIG. 2. The function $\varphi(x)$ of Eq. (16).

$$\left(\frac{y}{\sinh y}\right)^2 = 4y^2\sum_{n=1}^\infty n e^{-2ny}, \quad (13)$$

we can also relate $\Delta\sigma_{\mathrm{ee}}$ to the spectral determinant. We obtain

$$\Delta\sigma_{\mathrm{ee}} = -\lambda_\sigma \frac{e^2}{\pi \mathrm{Vol}}\sum_{n=1}^\infty \frac{1}{n}\left[\gamma^2\frac{\partial^3}{\partial\gamma^3}\ln S(\gamma)\right]_{\gamma=2n\pi/L_T^2}, \quad (14)$$

which is the central result of this paper. It is the starting point of the discussion below.

Application to the case of the square network. We have to compute $\gamma^2\frac{\partial^3}{\partial\gamma^3}\ln S(\gamma)$. We start from Eq. (6) and compute its second derivative. We obtain after some algebra

$$\Delta\sigma_{\mathrm{ee}} = -\lambda_\sigma \frac{e^2}{h}\frac{a}{8}\sum_{n=1}^\infty \frac{1}{n}\varphi\left(\sqrt{2n\pi}\frac{a}{L_T}\right), \quad (15)$$

where the function $\varphi(x)$ is given by

$$\varphi(x) = -\frac{8}{x^2} + \frac{2x\cosh x}{\sinh^3 x} + \frac{3}{\sinh^2 x} + \frac{3\coth x}{x} + \frac{2}{\pi}\left\{\left[\frac{3\tanh x}{x} - 3\right]K\left(\frac{1}{\cosh x}\right) + \left[3 - \frac{2x}{\sinh 2x}\right]E\left(\frac{1}{\cosh x}\right)\right\}, \quad (16)$$

$E(x)$ being the complete elliptical integral of the second kind.[30] The function $\varphi(x)$ is plotted in Fig. 2 and its limiting behaviors are easily obtained:[30]

$$\varphi(x) = \frac{4}{\pi} + O(x^2) \quad \text{for } x \to 0 \quad (17a)$$

$$= \frac{6}{x} - \frac{8}{x^2} + O(xe^{-2x}) \quad \text{for } x \to \infty. \quad (17b)$$

The L_T dependence of AA correction on a square network is displayed in Fig. 3, where we have plotted $\Delta\sigma_{\mathrm{ee}}(L_T)$ given by

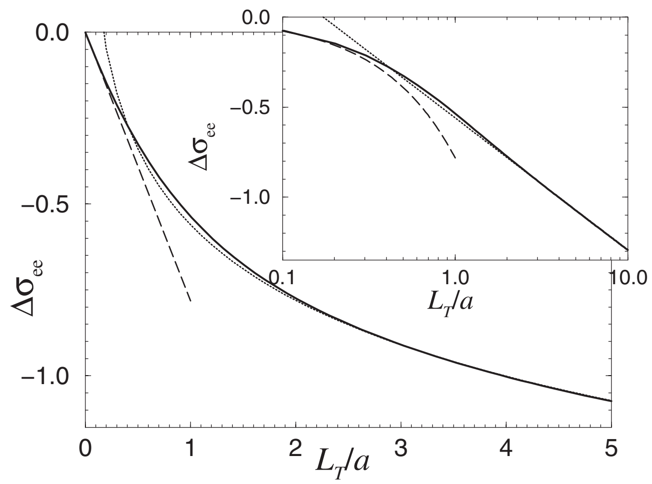

FIG. 3. The continuous line is $\Delta\sigma_{\rm ee}$ in unit of $\lambda_\sigma \frac{e^2}{h}$ as a function of L_T/a [the series (15) is computed numerically]. The dashed line is the 1D limit [Eq. (18)] and the dotted curve is the 2D limit [Eq. (19)].

Eq. (15). The dimensional crossover now occurs by tuning the ratio L_T/a. We consider the two limits.

1D *limit*. For $L_T \ll a$, we can replace the expansion (17b) in the series (15) Therefore

$$\Delta\sigma_{\rm ee} \simeq -\lambda_\sigma \frac{e^2}{h}\left[\frac{3\zeta(3/2)}{4\sqrt{2\pi}} L_T - \frac{\pi}{12}\frac{L_T^2}{a}\right], \tag{18}$$

with $\frac{3\zeta(3/2)}{4\sqrt{2\pi}} \simeq 0.782$. The dominant term again coincides with that for a connected wire,[8,10,11] while the second differs by a factor 2, as for the WL [see discussion after Eq. (7)].

2D *limit*. In the limit $L_T \gg a$, we introduce $\mathcal{N} = (L_T/a)^2$ and cut the sum (15) in two pieces: $\Sigma_1^\infty = \Sigma_1^{\mathcal{N}} + \Sigma_{\mathcal{N}}^\infty$. It is clear from the limit behaviors of $\varphi(x)$ that the first sum diverges logarithmically with $\mathcal{N}$, while the second brings a negligible contribution of order $\mathcal{N}^0$. Therefore

$$\Delta\sigma_{\rm ee} \simeq -\lambda_\sigma \frac{e^2}{h} a\left[\frac{1}{\pi}\ln\left(\frac{L_T}{a}\right) + C_{\rm ee}\right]. \tag{19}$$

The constant is estimated numerically. We find $C_{\rm ee} \simeq 0.56$.

Equations (18) and (19) should be divided by the cross section s of the wires.

The two functions $\Delta\sigma_{\rm WL}(\mathcal{B}=0, L_\varphi)$ (Fig. 1) and $\Delta\sigma_{\rm ee}(L_T)$ (Fig. 3) are very similar. Apart from the prefactors $2e^2/h$ and $\lambda_\sigma e^2/h$ which account, respectively, for the spin degeneracy and the interaction strength, the linear behaviors at the origin have different slopes (1 and 0.782) and the logarithmic behaviors are slightly shifted: $C_{\rm WL} \simeq 0.61$ and $C_{\rm ee} \simeq 0.56$.

IV. COMPARISON WITH EXPERIMENTS

The AA correction has been recently measured by Mallet *et al.*[34] in networks of silver wires with 3×10^4 and 10^5 cells, lattice spacing $a = 0.64\ \mu$m, and diffusion constant $D \simeq 100\ {\rm cm^2/s}$. The diffusion constant D has been measured separately (through measurement of the Drude conductance), therefore we can compare our result (15) with experiment using one fitting parameter only: the interaction parameter λ_σ. The 2D logarithmic behavior (19) has been observed in the range 100 mK $< T <$ 1 K from which the value $\lambda_\sigma^{\rm exp} \simeq 3.1$ was extracted, in agreement with similar measurements performed on a long silver wire for which[34,35] $\lambda_\sigma^{\rm exp,wire} \simeq 3.2$. We now compare with the theoretical value: for silver, Fermi wavelength is $k_F^{-1} = 0.083$ nm and Thomas-Fermi screening length $\kappa^{-1} = 1/\sqrt{8\pi\rho_0 e^2} = 0.055$ nm. In the Thomas-Fermi approximation, the parameter F is given by[11] $F = (\frac{\kappa}{2k_F})^2 \ln[1 + (\frac{2k_F}{\kappa})^2]$, therefore $F \simeq 0.58$. Using the 1D nonperturbative expression[8] $\lambda_\sigma = 4 + (48/F)(\sqrt{1+F/2} - 1 - F/4)$, we get $\lambda_\sigma^{\rm th} \simeq 3.24$, close to the experimental value.

V. CONCLUSION

Equations (14) and (15) are our main results. The first one shows that AA and WL can be formally related:

$$\Delta\sigma_{\rm ee}(L_T) = \frac{\lambda_\sigma}{2}\sum_{n=1}^{\infty}\frac{1}{n}\left[\gamma^2 \frac{\partial^2}{\partial\gamma^2}\Delta\sigma_{\rm WL}(L_\varphi)\right]_{\gamma \equiv 1/L_\varphi^2 = 2n\pi/L_T^2}. \tag{20}$$

This relation is reminiscent of the relation between WL and conductivity fluctuations (UCFs).[11,28,29,36] The main outcome of the relation initially derived in Ref. 36 was that both WL and UCF are governed by the same length scale L_φ. Here, we insist again on the fact that the AA correction is independent of the phase coherence length L_φ, which should only be understood in Eq. (20) as a formal parameter within the substitution $L_\varphi \to L_T/\sqrt{2n\pi}$. Finally, we point out that the validity of the relations (14) and (20) is the same as for Eqs. (1a) and (1b): the system should be such that it is meaningful to average uniformly the nonlocal conductivity $\sigma(r, r')$ to get the local conductivity $\sigma = \int \frac{dr dr'}{\rm Vol}\sigma(r, r')$. A similar discussion has been proposed to relate WL and conductivity fluctuations (Appendix E of Ref. 29).

Our starting point (10) is a formulation in the Fourier space, which implicitly assumes translation invariance. Whereas this assumption seems reasonable for a large regular network such as the square network studied in this paper, its validity is not clear for networks of arbitrary topology, which would need further development.

We have computed the AA correction in a large square network and shown that the result interpolates between the 1D [Eq. (18)] and 2D results, [Eq. (19)]. Interestingly, the 2D limit in a network involves a 1D constant $\lambda_\sigma^{\rm network} \simeq 4 - \frac{3}{2}F$, which is confirmed by experiments, as discussed in Sec. IV.

One interest of the network compared to the plane is to control the constant $C_{\rm ee}$ of Eq. (19): for a plane, a cutoff must be introduced in Eq. (12) at short time $t \sim \tau_e$ and the constant $C_{\rm ee}$ is replaced by a number that depends on the precise cutoff procedure. Note, however, that a measurement of the constant $C_{\rm ee}$ is limited by the determination of the lattice spacing a (due to the finite width of the wires, for example) and the diffusion constant D; an uncertainty δa on the lattice spacing and δD on the diffusion constant would yield an additional uncertainty $\frac{1}{\pi}(\frac{\delta a}{a} + \frac{\delta D}{2D})$ for $C_{\rm ee}$.

Experimentally, it would be interesting to observe the crossover from Eq. (18) and (19) by varying L_T/a. This was not possible in experiments of Mallet *et al.*[34] described in Sec. IV because measurements are complicated by the fact that electron-phonon interaction also brings a temperature-dependent contribution, $\Delta\sigma_{\text{e-ph}}$, at high temperature (above a few Kelvin). The conductivity is given by $\sigma=\sigma_0+\Delta\sigma_{\text{WL}}+\Delta\sigma_{\text{ee}}+\Delta\sigma_{\text{e-ph}}$. The WL can be suppressed by a magnetic field; however, the electron-phonon contribution is difficult to separate from $\Delta\sigma_{\text{ee}}$. Therefore the network should be patterned in a way such that the crossover 1D-2D remains below $T\sim 1$ K, where $\Delta\sigma_{\text{e-ph}}$ is negligible. As an example, we consider the silver networks studied in Ref. 21, for which $L_T=0.27\times T^{-1/2}$ (L_T in micrometer and T in kelvin). In order to see clearly the 1D and 2D regimes, it would be convenient to study two networks with different lattice spacings. If temperature is constrained by 10 mK$<T<$1 K, for $a=0.5$ μm we have $0.5\lesssim L_T/a\lesssim 5$, which probes the 2D regime over one decade. A second lattice with $a\sim 5$ μm would allow to probe the 1D regime since, in this case, $0.05\lesssim L_T/a\lesssim 0.5$.

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[1] B. L. Altshuler, A. G. Aronov, and D. E. Khmelnitsky, J. Phys. C **15**, 7367 (1982).

[2] S. Chakravarty and A. Schmid, Phys. Rep. **140**, 193 (1986).

[3] B. L. Al'tshuler and A. G. Aronov, Sov. Phys. JETP **50**, 968 (1979).

[4] B. L. Altshuler, D. Khmel'nitzkiĭ, A. I. Larkin, and P. A. Lee, Phys. Rev. B **22**, 5142 (1980).

[5] B. L. Altshuler and A. G. Aronov, Solid State Commun. **46**, 429 (1983).

[6] A. M. Finkel'shteĭn, Sov. Phys. JETP **57**, 97 (1983).

[7] C. Castellani, C. Di Castro, P. A. Lee, and M. Ma, Phys. Rev. B **30**, 527 (1984).

[8] B. L. Altshuler and A. G. Aronov, in *Electron-electron Interactions in Disordered Systems*, edited by A. L. Efros and M. Pollak (North-Holland, Amsterdam, 1985), p. 1.

[9] P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985).

[10] I. L. Aleiner, B. L. Altshuler, and M. E. Gershenson, Waves Random Media **9**, 201 (1999).

[11] É. Akkermans and G. Montambaux, *Mesoscopic Physics of Electrons and Photons* (Cambridge University Press, Cambridge, 2007).

[12] It is worth mentioning that a similar effect exists at higher temperature, in the ballistic regime ($\tau_{\text{ee}}\ll\tau_e$); Ref. 13 provides a nice review on this point and describes the crossover between the two regimes.

[13] G. Zala, B. N. Narozhny, and I. L. Aleiner, Phys. Rev. B **64**, 214204 (2001).

[14] A. E. White, M. Tinkham, W. J. Skocpol, and D. C. Flanders, Phys. Rev. Lett. **48**, 1752 (1982).

[15] P. M. Echternach, M. E. Gershenson, H. M. Bozler, A. L. Bogdanov, and B. Nilsson, Phys. Rev. B **50**, 5748 (1994).

[16] F. Pierre, A. B. Gougam, A. Anthore, H. Pothier, D. Esteve, and N. O. Birge, Phys. Rev. B **68**, 085413 (2003).

[17] C. Bäuerle, F. Mallet, F. Schopfer, D. Mailly, G. Eska, and L. Saminadayar, Phys. Rev. Lett. **95**, 266805 (2005).

[18] B. Pannetier, J. Chaussy, R. Rammal, and P. Gandit, Phys. Rev. B **31**, 3209 (1985).

[19] G. J. Dolan, J. C. Licini, and D. J. Bishop, Phys. Rev. Lett. **56**, 1493 (1986).

[20] M. Ferrier, L. Angers, A. C. H. Rowe, S. Guéron, H. Bouchiat, C. Texier, G. Montambaux, and D. Mailly, Phys. Rev. Lett. **93**, 246804 (2004).

[21] F. Schopfer, F. Mallet, D. Mailly, C. Texier, G. Montambaux, L. Saminadayar, and C. Bäuerle, Phys. Rev. Lett. **98**, 026807 (2007).

[22] B. Douçot and R. Rammal, Phys. Rev. Lett. **55**, 1148 (1985).

[23] B. Douçot and R. Rammal, J. Phys. (Paris) **47**, 973 (1986).

[24] M. Pascaud and G. Montambaux, Phys. Rev. Lett. **82**, 4512 (1999).

[25] Pascaud and Montambaux have rather considered thermodynamic properties. The nonlocal effects in networks have been further investigated in Ref. 27.

[26] E. Akkermans, A. Comtet, J. Desbois, G. Montambaux, and C. Texier, Ann. Phys. (N.Y.) **284**, 10 (2000).

[27] C. Texier and G. Montambaux, Phys. Rev. Lett. **92**, 186801 (2004).

[28] T. Ludwig and A. D. Mirlin, Phys. Rev. B **69**, 193306 (2004).

[29] C. Texier and G. Montambaux, Phys. Rev. B **72**, 115327 (2005); **74**, 209902(E) (2006).

[30] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, 5th eds. (Academic, New York, 1994).

[31] A. Cassam-Chenai and B. Shapiro, J. Phys. I **4**, 1527 (1994).

[32] The formula (5.1) of Ref. 8 has the wrong sign.

[33] This interaction assumes that the screening length is smaller than the transverse size of the wire.

[34] F. Mallet *et al.* (unpublished).

[35] L. Saminadayar, P. Mohanty, R. A. Webb, P. Degiovanni, and C. Bäuerle, Physica E (to be published).

[36] I. L. Aleiner and Ya. M. Blanter, Phys. Rev. B **65**, 115317 (2002).