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Journal of Magnetism and Magnetic Materials ■ (■■■■) ■■■–■■■

Journal of magnetism and magnetic materials

www.elsevier.com/locate/jmmm

Incomplete Andreev reflection in a clean SFS junction

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Abstract

We study the stationary Josephson effect in a ballistic superconductor/ferromagnet/superconductor junction for arbitrarily large spin polarizations. Due to the exchange interaction in the ferromagnet, the Andreev reflection is incomplete. We describe how this effect modifies the Josephson current in the crossover from a superconductor/normal metal/superconductor junction to a superconductor/half metal/superconductor junction.

PACS: 74.50.+r; 72.25.−b

Keywords: Superconductivity; Magnetism; Josephson effect

In the past, the Josephson effect in superconductor/ferromagnet/superconductor (SFS) junctions has mainly been studied for small spin polarizations. The Josephson current is due to the Andreev [1] conversion of singlet Cooper pairs into correlated electrons and holes with opposite spins, which propagate coherently in the ferromagnetic metal. Applying the Eilenberger equations [2] to a clean multichannel SFS junction, Buzdin et al. [3] have predicted that this non dissipative current oscillates as a function of both the exchange energy splitting E_{ex} and the length d of the ferromagnet, because of the mismatch between spin-up and spin-down Fermi wavevectors $2E_{\mathrm{ex}}/(\hbar v_{\mathrm{F}})$. This quasiclassical result assumes that the Andreev reflection is complete, as it is fully justified for weakly spin-polarized ferromagnetic alloys. This assumption is incorrect for devices with high spin polarization which are used to manipulate spin-polarized currents [4]. In the recently discovered half metals (HM) such as CrO_2, the current is completely spin-polarized because one spin subband is insulating. Strong ferromagnetic elements like Fe, Co or Ni, also exhibit quite large spin polarizations [5,6]. Therefore, it is important to revisit the physics of the SFS Josephson effect *for arbitrarily large spin polarizations*. Moreover first experimental evidence for oscillating critical currrent have been recently reported in Nb–Ni–Nb junctions [7] and in Nb–FeNi–Nb junctions [8].

In Section 1, we first consider a purely one-dimensional clean ferromagnet connected between two superconducting leads. The excitation spectrum and the current are obtained for arbitrary large spin polarizations $\eta = E_{\mathrm{ex}}/E_{\mathrm{F}}$ using the Bogoliubov–de Gennes equations. The probability for Andreev reflection decreases abruptly when E_{ex} approaches the Fermi energy E_{F} [9]. Then, the Andreev scattering is replaced by normal reflection of electrons and the Josephson current vanishes. In Section 2, we consider the more realistic case of a multichannel SFS junction with a finite section. As the exchange field is increased, the Andreev reflection is suppressed for electrons propagating with a large incidence, so that the number of channels contributing to the total current decreases. For large spin polarizations, we find that the current depends *separately* on the product $k_{\mathrm{F}}d$ and on the spin polarization η. The oscillations of the critical current are *reduced and shifted* comparatively to the predictions of the quasiclassical

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0304-8853/$ - see front matter
doi:10.1016/j.jmmm.2005.10.040

theory [3] in which only the single parameter $2E_{\mathrm{ex}}d/(\hbar v_{\mathrm{F}}) = \eta k_{\mathrm{F}}d$ is relevant. For small spin polarizations, we naturally recover the quasiclassical results. In the opposite limit of a half metal $E_{\mathrm{ex}} \to E_{\mathrm{F}}$, the critical current tends to zero because the Andreev reflection is totally suppressed for all the transverse channels. Our results are in agreement with those of Radovic et al. [10] although they are not derived in the same way.

1. Purely one-dimensional SFS junction

We consider a purely one-dimensional ballistic ferromagnet with length d connected between two superconducting leads.

1.1. Model

The itinerant ferromagnetism is described within the Stoner model by an effective potential $V_\sigma = V_\sigma(x) = -\sigma E_{\mathrm{ex}}$, which depends on the spin direction $\sigma = \pm 1$. The superconducting pair potential is $\Delta(x) = |\Delta| \mathrm{e}^{\mathrm{i}\chi/2}$ in the left lead and $\Delta(x) = |\Delta| \mathrm{e}^{-\mathrm{i}\chi/2}$ in the right one. In the absence of spin-flip scattering, the Bogoliubov–de Gennes, equations split in two sets of independent equations for the spin channels $(u_\uparrow, v_\downarrow)$ and $(u_\downarrow, v_\uparrow)$

$$\begin{pmatrix} H_o + V_\sigma & \Delta(x) \\ \Delta(x)^* & -H_o^* + V_\sigma \end{pmatrix} \begin{pmatrix} u_\sigma \\ v_{-\sigma} \end{pmatrix} = \varepsilon \begin{pmatrix} u_\sigma \\ v_{-\sigma} \end{pmatrix}, \tag{1}$$

where $\varepsilon = \varepsilon(\chi)$ is the quasiparticle energy measured from the Fermi energy $E_{\mathrm{F}} = \hbar^2 k_{\mathrm{F}}^2/2m$. The kinetic part of the Hamiltonian is $H_o = [(-\mathrm{i}\hbar d/\mathrm{d}x - qA(x))^2 - E_{\mathrm{F}}]/2m$ where m is the effective mass of electrons and holes. The vector potential $A(x)$ is responsible for the phase difference χ between the leads.

1.2. Eigenvalue equation

In the ferromagnet, the eigenvectors of Eq. (1) are electrons and holes with plane wave spatial dependencies. The electron and hole longitudinal wavevectors, denoted respectively $k^{\sigma}_{\varepsilon,\eta}$ and $h^{-\sigma}_{\varepsilon,\eta}$, satisfy

$$\frac{\hbar^2 [k^{\sigma}_{\varepsilon,\eta}]^2}{2m} - E_{\mathrm{F}} = \varepsilon + \sigma E_{\mathrm{ex}},$$

$$\frac{\hbar^2 [h^{-\sigma}_{\varepsilon,\eta}]^2}{2m} - E_{\mathrm{F}} = -\varepsilon - \sigma E_{\mathrm{ex}}. \tag{2}$$

Matching the wavefunctions and their derivatives at the FS interfaces, we obtain the following eigenvalue equation for the Andreev levels [11]

$$\begin{aligned} 16kh\cos\chi = & -2(k^2 - k_{\mathrm{F}}^2)(h^2 - k_{\mathrm{F}}^2)[\cos\Delta kd - \cos\Sigma kd] \\ & - (k - k_{\mathrm{F}})^2 (h + k_{\mathrm{F}})^2 \cos(\Sigma kd + 2\varphi_\varepsilon) \\ & - (k + k_{\mathrm{F}})^2 (h - k_{\mathrm{F}})^2 \cos(\Sigma kd - 2\varphi_\varepsilon) \\ & + (k + k_{\mathrm{F}})^2 (h + k_{\mathrm{F}})^2 \cos(\Delta kd - 2\varphi_\varepsilon) \\ & + (k - k_{\mathrm{F}})^2 (h - k_{\mathrm{F}})^2 \cos(\Delta kd + 2\varphi_\varepsilon), \end{aligned} \tag{3}$$

where for convenience, we define $k = k^{\sigma}_{\varepsilon,\eta}$, $h = h^{-\sigma}_{\varepsilon,\eta}$, $\Delta k = \Delta k^{\sigma}_{\varepsilon,\eta} = k - h$, $\Sigma k = \Sigma k^{\sigma}_{\varepsilon,\eta} = k + h$ and $\varphi_\varepsilon = \arccos(\varepsilon/\Delta)$. There are four typical energies in this problem: the superconducting gap Δ, the exchange energy E_{ex}, the level spacing $\min(\hbar v_{\mathrm{F}}/d, \Delta)$ and the Fermi energy E_{F}. As seen from Eq. (3), the exact spectrum $\varepsilon^\sigma(\chi)$ depends on two parameters: the spin polarization $\eta = E_{\mathrm{ex}}/E_{\mathrm{F}}$ and the product $k_{\mathrm{F}}d$, whereas it depends only on the single combination $\eta k_{\mathrm{F}} d$ in the quasiclassical approximation. In the present work, the spin polarization $\eta = E_{\mathrm{ex}}/E_{\mathrm{F}}$ is arbitrary and the ratio $\Delta/E_{\mathrm{F}} \ll 1$.

1.3. Spectrum and current

For small spin polarizations $\eta \to 0$, we recover the spectrum of Ref. [12]

$$\cos\chi = \cos\left(\frac{2\varepsilon d}{\hbar v_{\mathrm{F}}} + a - 2\varphi_\varepsilon\right), \tag{4}$$

where the parameter $a = (\sqrt{1+\eta} - \sqrt{1-\eta})k_{\mathrm{F}}d$ is the phase shift accumulated between an electron and a hole located at the Fermi level during their propagation on a length d. For a finite spin polarization η, we have shown in Ref. [11] that *gaps open at* $\chi = 0$ *and* $\chi = \pi$, see Figs. 1(a,b). Except for these gaps and up to large spin polarizations, the spectrum is identical to the one given by the Eq. (4) assuming complete Andreev reflection, and the current is practically unaffected (Fig. 1(d)). However, the spectrum undergoes a qualitative change above a particular spin polarization η^*: a gap opens at the Fermi level as shown in Fig. 1(c) and the current has no discontinuity anymore

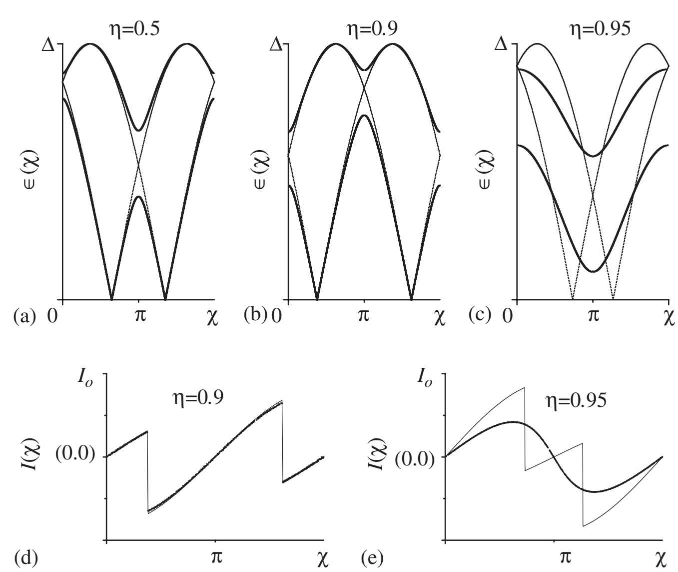

Fig. 1. Spectrum and current of a short SFS junction for increasing spin polarizations η with $k_{\mathrm{F}}d = 10$ obtained from solving Eq. 3. The thin lines represent the corresponding quasiclassical estimations. $I_o = 2\pi\Delta/\phi_o$.

(Fig. 1(e)). The region $\eta^* < \eta < 1$ in which Andreev reflection and ordinary reflection coexist is extremely small and scales as $1/(k_F d)^2$. As a result, the Josephson current through a single channel SFS junction is given to great accuracy by the formula with perfect Andreev reflection [3]

$$\mathrm{i}(\chi, k_F d, \eta, \theta_n = 0) = \frac{\pi \Delta}{\phi_o} \sum_{\sigma=\pm 1} \sin \frac{\chi + \sigma a}{2} \times \tanh\left(\frac{\Delta}{2T} \cos\left(\frac{\chi + \sigma a}{2}\right)\right), \quad (5)$$

for $\eta < \eta^* \approx 1$ and it is zero for $\eta > 1$. In Eq. (5), $\phi_o = h/e$ is the magnetic flux quantum.

2. Multichannel SFS junction

We consider now a ballistic SFS junction with a finite width [13]. The transverse channels are labelled by the incidence angle θ_n. An electron cannot find a hole to form an Andreev bound state if its transverse energy $E_n = E_F \sin^2\theta_n > E_F - E_{ex}$. Thus for angle $\theta_n > \theta_\eta = \arccos\sqrt{\eta}$, the electron is normally reflected as an electron with the same spin. Such a process is insensitive to the superconducting phase and thus carry no Josephson current. In the opposite case $\theta_n \ll \theta_\eta$, the Andreev reflection is complete and supports a finite current. In the following, the former kind of channel is referred as "Andreev inactive" and the latter as "Andreev active". Generalizing the result of Section 1.3, we obtain that the crossover between Andreev active and inactive channels occurs in a narrow window of incidences at vicinity of $\theta_\eta = \arccos\sqrt{\eta}$. Below this cut-off, the current carried by a single Andreev active channel is

$$\mathrm{i}(\chi, k_F d, \eta, \theta_n) = \frac{\pi \Delta}{\phi_o} \sum_{\sigma=\pm 1} \sin \frac{\chi + \sigma a_n}{2} \times \tanh\left(\frac{\Delta}{2T} \cos\left(\frac{\chi + \sigma a_n}{2}\right)\right), \quad (6)$$

and it is zero for $\theta_\eta > \arccos\sqrt{\eta}$. Treating large exchange splitting requires to take into account the exact band structure. For an isotropic parabolic band, the phase shift between an electron and its Andreev reflected hole is

$$a_n = k_F d \cos\theta_n \left(\sqrt{1 + \frac{\eta}{\cos^2\theta_n}} - \sqrt{1 - \frac{\eta}{\cos^2\theta_n}}\right). \quad (7)$$

The total current is the sum of the currents carried by each of the Andreev active levels

$$I(\chi, k_F d, \eta) = \frac{k_F^2 S}{2\pi} \int_0^{\theta_\eta} \mathrm{d}\theta \sin\theta \cos\theta \; \mathrm{i}(\chi, k_F d, \eta, \theta), \quad (8)$$

where S is the cross section area of the ferromagnet. This expression, together with Eqs. (6) and (7), gives the Josephson current $I(\chi, k_F d, \eta)$ of a clean SFS junction in *for arbitrarily large spin polarization*. Fig. 2 represents the critical current as a function of the spin polarization η, extrapolating from a SNS junction ($\eta = 0$) to a S/HM/S junction ($\eta = 1$). The local minima of the critical current correspond to $0 - \pi$ transitions [10]. In the limit of small polarization $\eta = E_{ex}/E_F \to 0$, we recover the quasiclassical current-phase relationship [3] in which all the transverse channels contribute because $\theta_\eta \to \pi/2$. When η is increased, the oscillations of the critical current as a function of the length d deviate from the expectations of the quasiclassical calculation, as shown in Fig. 3. The effect of a finite temperature on the critical current has been considered in Ref. [13].

In conclusion, we have obtained the Josephson current in a short ballistic SFS junction in the range of large spin polarizations which is relevant for spintronics materials. The physical effect involved is the suppression of Andreev reflection as the exchange energy is increased. The active levels with small incidence are essentially unaffected by the

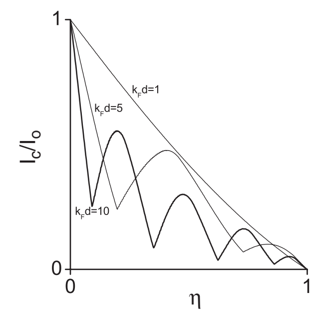

Fig. 2. Zero temperature critical current I_c as a function of $\eta = E_{ex}/E_F$ for different lengths of the ferromagnet $k_F d = 1, 5, 10$. $I_o = \pi\Delta_0/(eR_N)$ with $R_N = h/(2e^2 M)$ and $M = k_F^2 S/4\pi$.

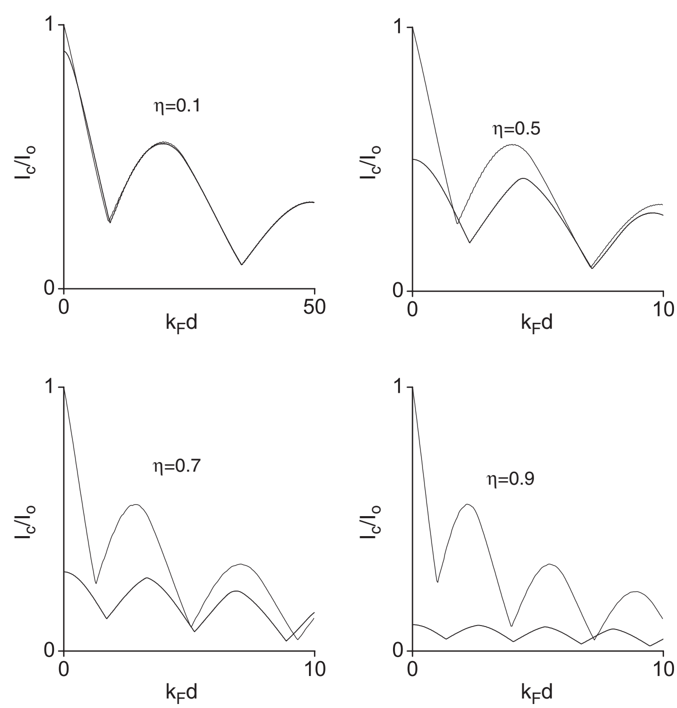

Fig. 3. Zero temperature critical current I_c as a function of $k_F d$ (thick lines), for different values of η. As the spin polarization increases, the exact current deviates from the quasiclassical estimate (thin lines). $I_o = \pi\Delta/(eR_N)$.

ordinary reflection whereas levels with high incidence do not carry any current.

We thank Zoran Radovic for useful discussions and Igor Zutic for his comments.

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