Comment on "Level Statistics of Quantum Dots Coupled to Reservoirs"

In a recent Letter, König *et al.* describe how two levels of a quantum dot are broadened by the coupling to an external reservoir [1]. To do this they consider that these two levels of the dot are nearest neighbors in energy and are well separated from other levels. They find that when the coupling to the reservoir increases, the spectral weight of one of these two states σ, after being roughly a Lorentzian when the width Γ_σ is smaller than the distance $\Delta\epsilon$ between the two levels, evolves into a structure made of a sharp peak located at the center of the two levels plus a broad contribution. Since each peak moves towards the middle of the original levels, this motion is analyzed in terms of level attraction. We show here that the conclusions of Ref. [1] are very specific to the case of two levels. Considering that more than two levels lead to a different description, namely, each spectral weight is actually split into *several peaks* so that it is meaningless to analyze the results in terms of level attraction. We consider the same Hamiltonian as in Ref. [1]

$$H = \sum_k \epsilon_k a_k^+ a_k + \sum_{\sigma=1,m} \epsilon_\sigma a_\sigma^+ a_\sigma + \sum_{k\sigma} [T_{k\sigma} a_k^+ c_\sigma + \text{H.c.}]. \tag{1}$$

This generalizes the Hamiltonian considered in Ref. [1] to m discrete levels coupled to a continuum. The spectral weight of a given level σ is given by $A_\sigma(\omega) = -\text{Im}G_{\sigma\sigma}(\omega + i0^+)/\pi$. The Green's function $G_{\mu\sigma}$ obey the $m \times m$ matrix equation: $\sum_\mu M_{\lambda\mu} G_{\mu\sigma} = \delta_{\lambda,\sigma}$ where $M_{\lambda\mu} = (\omega - \epsilon_\lambda)\delta_{\lambda\mu} - \Sigma_{\lambda\mu}$, with $\Sigma_{\lambda\mu} = -i\pi \sum_k T_{k\lambda} T^*_{k\mu} \delta(\omega - \epsilon_k)$. Since the density of states in the reservoirs forms a continuum, $\Sigma_{\lambda\mu}$ can be rewritten as $\Sigma_{\lambda\mu} = -i\pi \langle T_{k\lambda} T^*_{k\mu} \rangle \rho_0$, where the density of states of the continuum, ρ_0, is supposed to be constant. In the case considered in Ref. [1] where the tunnel matrix elements are assumed to be independent of the reservoir state, $T_{k\sigma} = T_\sigma$, the above matrix can be diagonalized and the Green's function can be calculated. We find

$$G_{\sigma\sigma}(\omega) = g_\sigma(\omega) \frac{2 + i\sum_{\lambda\neq\sigma} \Gamma_\lambda g_\lambda(\omega)}{2 + i\sum_\lambda \Gamma_\lambda g_\lambda(\omega)}, \tag{2}$$

where $\Gamma_\lambda = 2\pi |T_\lambda|^2 \rho_0$ is the width of a level without coupling between levels. $g_\lambda(\omega) = (\omega - \epsilon_\lambda)^{-1}$. The complex eigenvalues $\{\tilde\epsilon_\alpha + i\tilde\Gamma_\alpha\}$ are solutions of

$$\sum_\lambda \frac{\Gamma_\lambda/2}{\tilde\epsilon_\alpha + i\tilde\Gamma_\alpha - \epsilon_\lambda} = i. \tag{3}$$

When the Γ_λ increase and become large compared to the average distance between levels Δ, the spectral weight $A_\sigma(\omega)$ of each level is split into *a series* of $(m-1)$ peaks whose centers are given by $\sum_\lambda (\tilde\epsilon_\alpha - \epsilon_\lambda)^{-1} = 0$ and are placed at intermediate positions between the unperturbed levels [Figs. 1(a) and 1(b)]. Their widths $\tilde\Gamma_\alpha$ are given of the order of $[\sum_\lambda (\Gamma_\lambda/2)/(\epsilon_\lambda - \tilde\epsilon_\alpha)^2]^{-1} \sim \Delta^2/\Gamma$ where Γ is the typical value of a bare level. The corresponding spectral weights are $A_\sigma^\alpha = 1/[(\tilde\epsilon_\alpha - \epsilon_\sigma)^2 \sum_\lambda 1/(\epsilon_\lambda - \tilde\epsilon_\alpha)^2]$. The $(m-1)$ peaks actually carry $(1 - 1/m)$ of the spectral weight. The rest is carried by a very broad Lorentzian of width $\sum_\lambda \Gamma_\lambda$. When two neighboring levels are well separated from others the spectral weight tends to concentrated on only one peak, the situation considered in Ref. [1]. However, as it is shown in Fig. 1(c), the weight of other peaks may be not negligible.

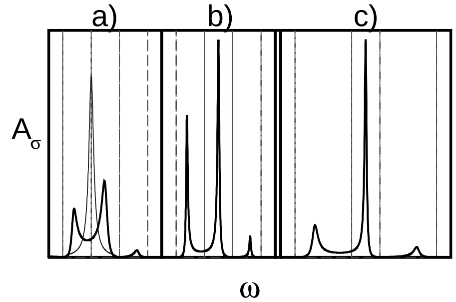

FIG. 1. Spectral weight $A_\sigma(\omega)$ for the 2nd out of four discrete levels whose unperturbed energies are figured by vertical lines. All the widths Γ_α are taken equal to $\Gamma_\sigma = \Gamma$. (a) $\Gamma/\delta = 0.1$ (thin lines); $\Gamma/\delta = 1$. (b) $\Gamma/\delta = 3$. (c) $\Gamma/\delta = 3$; here the bare levels are not equidistant $(2\delta, \delta, 2\delta)$. δ is the distance between the 2nd and the 3rd levels.

Finally we note that this structure is extremely sensitive to the type of coupling to the reservoirs. References [1,2] assume a constant coupling $T_{k\sigma} = T_\sigma$. The most general case $\langle T_{\lambda_i k} T_{k\lambda_j} \rangle \neq \langle T_{\lambda_i k} \rangle \langle T_{k\lambda_j} \rangle$ has also been discussed in Ref. [2]. When the couplings to the reservoir levels are uncorrelated and symmetric so that $\langle T_{\lambda_i k} T_{k\lambda_j} \rangle = \langle T_{\lambda_i k} \rangle \langle T_{k\lambda_j} \rangle = 0$ for $i \neq j$, the off-diagonal elements in the matrix M vanish and one finds that each level coupled to the continuum is broadened into a Lorentzian according to the Fermi golden rule; neither its center nor its width are altered: "The resonances do not talk to each other." The coupling between resonances is thus critically dependent on the type of coupling with the reservoirs.

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Received 2 February 1999
PACS numbers: 73.40.Gk, 73.23.–b, 85.30.Vw

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